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materials of an older building, which were used in the building of the church now in ruins. In the same way one of the figures in the Museum, from the Dawson collection, appears to have been built into the wall of the church where it was found. Under such circumstances, the actual antiquity of these curious figures is quite problematical. The subject is a new one, and well deserving of the attention of antiquaries, to whom this notice is submitted more as a suggestion for consideration than as an opinion. The number of facts known are few, and probably it may be premature to attempt a generalization.

April 22.

SIR WM. R. HAMILTON, LL. D., President, in the Chair.

READ,—a letter from the Secretary of the Lord Lieutenant, presenting to the Academy the stones containing the inscription from the old bridge of Athlone.

RESOLVED,—That the thanks of the Academy be given to His Excellency the Lord Lieutenant for his donation.

The Rev. Professor Graves read a paper on the Algebraic Geometry of Curves traced upon given Surfaces.

Let $U = \phi\,(x, y, z) = 0$ be the equation of a surface referred to ordinary rectangular coordinates. Its complete differential will be

$$Pdx + Qdy + Rdz = 0.$$

Making

$$X = \frac{P}{R}, \text{ and } Y = \frac{Q}{R},$$

Mr. Graves denominates X and Y the *normal coordinates* of a point on the surface. When they are known, the x, y, z of the point are determined by the three equations

$$\mathrm{U} = 0, \frac{\mathrm{P}}{\mathrm{R}} = \mathrm{X}, \text{ and } \frac{\mathrm{Q}}{\mathrm{R}} = \mathrm{Y}.$$

As P, Q, R, are proportional to the cosines of the angles which the normal at the point (X, Y) makes with the axes; it is easy to shew that if we describe a sphere with its centre at the origin and radius $= 1$, X and Y will be at the same time the rectangular spherical coordinates of that point on the sphere at which the tangent plane is parallel to the plane touching the surface $\mathrm{U}=\mathrm{o}$ at the point (X, Y). Thus, to every point on the latter corresponds a point on the former: and a succession of points, or a line of any kind, on the surface $\mathrm{U}=\mathrm{o}$ is in general represented by a succession of points, or a line upon the auxiliary sphere.

As plane curves have been classed according to the degrees of the equations by which they are represented, so curves traced on any given surface may be advantageously distinguished by the degrees of the equations in X and Y which define them. For the properties of a curve, traced on the surface $\mathrm{U}=\mathrm{o}$, and characterized by an equation of the *n*th degree between X and Y, are, so far as we regard only the relations of normals or tangent planes along it, identically the same as those of the spherical curve which has the same equation. But the analogy between spherical and plane curves of the *n*th degree has been already established. Instead, then, of looking upon the ***shortest lines*** on a surface as analogous to the right line, Mr. Graves directs his attention to the line defined by the equation

$$a\mathrm{X} + b\mathrm{Y} + 1 = 0, \qquad \text{(A)}$$

the geometrical character of which is, that the normal at any point on it is always parallel to a fixed plane. Systems of such lines upon any surface possess, in general, those properties of right lines which have been termed *projective*.

Thus, for instance: "If four lines of the first degree, diverging from the same point on a given surface, be cut in four points, a, b, c, d, by another line of the same kind, we shall have

$$\frac{\sin[a, d] . \sin[b, c]}{\sin[a, b] . \sin[c, d]} = \text{a constant,"}$$

$[a, b]$ being used to denote the angle between the normals to the surface at the points a and b.

The normals along the line (A) are all parallel to the tangent plane at the point whose normal coordinates are a and b. Mr. Graves designates this point *the pole* of the line. And if a and b be connected by an equation, so that the pole describes some curve of the nth degree, the line (A) will always touch another curve to which n tangent lines of the first degree may in general be drawn from the same point. This relation between the curves being obviously reciprocal, Mr. Graves calls them *reciprocal curves*. Here is laid the foundation of a theory of polar reciprocals for curves traced upon any given surface.

Amongst other exemplifications of this method, Mr. Graves employs it to discuss the lines of greatest and least curvature on the surface of an ellipsoid. Their equation in normal coordinates is

$$\frac{a^2}{(a^2 - k^2)} \text{X}^2 + \frac{b^2}{(b^2 - k^2)} \text{Y}^2 + \frac{c^2}{c^2 - k^2} = 0,$$

where a^2, b^2, c^2 are the squares of the semi-axes of the ellipsoid, and k^2 is indeterminate.

Now, from the mere fact of this equation being of the second degree, it follows that the sum or difference of the angles between the tangent plane at any point on a line of curvature and two fixed planes is constant.

But further, all the spherical curves of the second degree represented by the preceding equation are *biconfocal:* and it is easy to shew that their common foci are the points on the sphere which correspond to the umbilici of the ellipsoid. Hence, the sum or difference of the angles between the tangent plane at any point on a line of curvature, and the tangent planes at two umbilici, is constant; or, as the tangent planes at the umbilici are parallel to the planes of circular section, we have the following elegant theorem:

"The sum or difference of the angles between the tangent plane at any point along a line of curvature on an ellipsoid, and the two planes of circular section, is constant."

The proposition just mentioned was, it is believed, first published by Sir William Hamilton, in the Dublin University Review, part (3); the short article which contains it being dated June, 1833. It has also been published by Dr. Joachimsthal, in a paper printed in the 26th vol. of Crelle's Journal, and dated January, 1842, where that geometer claims it as "novum neque inelegans."

The reciprocal of the line of curvature has for its equation

$$\frac{a^2 - k^2}{a^2} \mathrm{X}^2 + \frac{b^2 - k^2}{b^2} \mathrm{Y}^2 + \frac{c^2 - k^2}{c^2} = 0,$$

in which if we make

$$\mathrm{X} = \frac{x}{z}, \text{ and } \mathrm{Y} = \frac{y}{z},$$

we shall get the equation of the cone, whose generatrices are parallel to the normals along the reciprocal of the line of curvature, and whose vertex is at the centre of the ellipsoid. After this substitution the last equation becomes

$$x^2 + y^2 + z^2 - k^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 0,$$

from the form of which it is evident that the cone passes through the intersection of the given ellipsoid, and a concentric sphere having k for its radius. The properties of this cone, and its relation to the lines of curvature, were first noticed by Professor Mac Cullagh.*

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* Proceedings of the Academy, vol. ii. p. 499.

For the quadrature of areas on the surface of the ellipsoid Mr. Graves gives the following formula:

$$\text{area} = a^2b^2c^2 \iint \frac{(1 + \mathrm{X}^2 + \mathrm{Y}^2)^{\frac{1}{2}}\, d\mathrm{X}\, d\mathrm{Y}}{(c^2 + a^2\mathrm{X}^2 + b^2\mathrm{Y}^2)^2}.$$

The President made some observations on the communication of Professor Graves.

Mr. Clibborn read a notice of certain points in Egyptian History.

Read,—The following Report from the Council:

"That the Council do recommend to the Academy, at its next meeting, to carry into effect the following recommendations of the Committee of Antiquities, and to request a vote of the Academy of £100 for the fitting-up of the Museum.

"1. That the resolution of the Academy, of the 30th of November, 1842, for making a new Board Room in the lower part of the house, be carried into effect.

"2. That the present Board Room be converted into a Museum.

"3. That, for this purpose, two tables be provided, to stand across the new Museum, opposite the piers. The present glass cases, except that containing the Cross of Cong, to be placed on those tables, and flat glazed cases to be added; the Cross of Cong to stand on a separate pedestal between the tables, *i. e.* in the centre of the room.

"The Council having taken into consideration the best means of exhibiting the antiquarian treasures contained in the Museum of the Academy, and thus giving proper importance and utility to that department, appointed a Committee, which recommended the adoption of the resolutions of the Committee of Antiquities. These have been adopted by the Council, and are now proposed for the adoption of the Academy."

Resolved,—That this Report and Recommendation of the Council be adopted, and that £100 be placed at the disposal of the Council for the purpose specified.